Helicon wave excitation and propagation in a toroidal heliac: Experiment and theory

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In this paper a comparison between theoretical calculations and experimental measurements of helicon wave coupling by a partial-turn shielded antenna in a toroidal heliac is presented. Despite the complicated heliac geometry, the wave dispersion, field profiles, and antenna radiation resistance are described well by a cylindrical uniform plasma theory. It also presents a simple formula relating the radiation resistance to the measured wave magnetic field per unit antenna current. Finally, the scaling of these results to the Heliac-1 (H-I) \cite{Fri 17, 123 (1990)}; a larger heliac now operating at the Australian National University (ANU), is discussed. © 1995 American Institute of Physics.

I. INTRODUCTION

The fast wave in the frequency range \( \omega_c < \omega < \omega_e \), referred to as the helicon or low-frequency whistler wave, has attracted considerable attention recently because of the possibility of efficiently producing a high-density plasma in low-field plasma devices.\textsuperscript{1-4} Efficient plasma formation and maintenance is also an important issue for stellarators, where a background plasma for heating and experimentation has to be produced.\textsuperscript{5-7} Fast waves are also important for the heating of plasmas in the ion-cyclotron range of frequencies (ICRF). The study of fast wave heating in the complex geometry of a stellarator is a topic of considerable interest given the success of ICRH in tokamaks.\textsuperscript{8-10}

Both these issues are currently under investigation in the Heliac-I\textsuperscript{11} (H-I); a three period helical axis stellarator or heliac recently commissioned at the Australian National University (ANU). The first issue to be addressed is the fast wave antenna coupling efficiency in heliac geometry. For this purpose we have performed experiments over a range of frequencies in the Small Heliac Experimental Apparatus\textsuperscript{12} (SHEILA), a small prototype toroidal heliac with the same geometry as H-I. We employ a partial-turn loop antenna typical of that used in ICRH experiments and compare the experimental measurements of the amplitude of the wave fields and antenna loading resistance with a simplified one-dimensional code for scaling to H-I. We demonstrate that, surprisingly, the geometry does not appear to significantly affect the dispersion of the fast wave. In cylindrical plasmas of similar dimensions to those of SHEILA, the \( m = +1 \) mode dominates the power spectrum of the radiated wave fields.\textsuperscript{1,2,4,13} This result is also confirmed in SHEILA, despite the complicated heliac geometry. We also demonstrate that it is possible to couple a significant proportion of the antenna power into the fast wave without a large parasitic loading.

The paper is structured as follows. In Sec. II we describe the antenna–wave coupling theory to be compared with the experimental results. In Sec. III we describe the experimental setup. In Sec. IV we present the experimental results and compare them with theory. In Sec. V we conclude with a discussion and a scaling of the results to H-I.

II. THEORY

The theory of helicon wave propagation has been discussed by many authors.\textsuperscript{14-16} We extend this work by calculating the coupling to helicon waves by a partial-turn loop antenna.

The helicon wave is a fast wave mode that propagates in the frequency range \( \omega_c < \omega < \omega_e \), where \( \omega_i \) and \( \omega_e \) are, respectively, the ion and electron cyclotron frequencies. The cold plasma tensor is given by

\[
\varepsilon = \begin{pmatrix}
\varepsilon_x & -i\varepsilon_k & 0 \\
-i\varepsilon_k & \varepsilon_x & 0 \\
0 & 0 & \varepsilon_l
\end{pmatrix}.
\]

(1)

If all terms in the dielectric tensor are retained, a slow wave referred to as the lower hybrid wave can also propagate in this frequency range. If we make the usual approximation that \( \varepsilon_k = 0 \), the tensor only describes the helicon wave. In the absence of damping, the remaining components of the dielectric tensor are given by

\[
\begin{align*}
\varepsilon_x &= \frac{\omega_p^2}{\omega \omega_e} - \frac{1}{1 - f_e^2}, \\
\varepsilon_{kl} &= 1 - \frac{\omega_p^2}{\omega_e^2},
\end{align*}
\]

(2)

where \( f_e = \omega / \omega_e \), and \( \omega_p \) is the electron plasma frequency.

We consider the plasma and antenna geometry depicted in Fig. 1. A partial-turn loop antenna of subtended angle \( \theta_A \) and axial width \( \Delta \) is immersed in a uniform magnetized plasma filled cylindrical waveguide of infinite length in the \( z \) direction. The interval \( a_1 < r < a_2 \) is the radial extent of the antenna, and a conducting wall is located at \( r = b \).

In this section we present the essentials of the theory. For the details, the reader is referred to the Appendix. With \( \varepsilon_k = 0 \) and in the absence of a source, the following differential equation is satisfied by \( B_z \):

\[
\frac{\partial^2 B_z}{\partial r^2} + \frac{1}{r} \frac{\partial B_z}{\partial r} + \left( \frac{T^2 - m^2}{r^2} \right) B_z = 0,
\]

(3)
where $m$ is the poloidal mode number, $k$ is the parallel wave number, and

$$T^2 = k^2 - k^2,$$

$$\beta^2 = \left( \frac{k^2 + \epsilon_i k_0^2}{\epsilon_i k_0^2 + \epsilon_k k^2} \right) \epsilon_i^2 k_0^2,$$

where $k_0 = \omega/c$. The dispersion relation obtained when the boundary condition $B_\perp(b) = 0$ is applied, is

$$m \alpha J_m(Tb) + kTbJ'_m(Tb) = 0,$$

where the parallel wave number is obtained by the solution of Eq. (4).

Equation (5) versus frequency is plotted in Fig. 2 in the range 1-30 MHz for the following conditions typical of SHEILA: $n_e = 8 \times 10^{18}$ m$^{-3}$, $B_0 = 0.14$ T, and $b = 0.053$ m. This value of $b$ is chosen to be the inner diameter of the toroidal field coils in SHEILA. The finite electron mass terms in the dielectric tensor have been shown to affect significantly the dispersion and damping of helicon waves in low-density plasmas. In Fig. 2 we have plotted two curves, one for which $f_e$ has been set to zero and $\epsilon_i^2$ has been set to infinity, so that $E_i = 0$ and one where these terms have been retained. As can be seen, electron mass effects are of little importance in this experiment.

The complicated exact expression for the antenna radiation resistance for the situation depicted in Fig. 1 is presented in the Appendix. It is of more use in practical situations, however, to present a general formula for the radiation resistance of waves propagating along an infinite cylinder, as obtained from the axial component of the Poynting flux in terms of the square of the measured wave magnetic field per unit antenna current. Such a quantity is independent of the nature of both the antenna and the boundary conditions, but requires a measurement of the wave magnetic field. It is given by

$$R_{rad} = \frac{\omega}{\mu_0 k} \left( \frac{\langle a^2 + k^2 \rangle \langle B^2 \rangle}{\langle \mathbf{I}^2 \rangle} \right) \frac{\pi b^2}{T_{ant}},$$

where $I_{ant}$ is the peak antenna current, and the symbol $\langle \rangle$ refers to a spatial average over the cylinder cross section. The $B$ appearing in (6) is the amplitude of the wave magnetic field radiated by the antenna. The experimentally measured value of $B$, however, depends on the axial location of the probe as a result of wave damping. Wave attenuation must therefore be taken into account in order to apply Eq. (6) to the experimentally measured $B$.

In order to compare the cylindrical wave field profiles with the measured profiles in heliac geometry, we calculate the equivalent cylindrical angles and radii as follows. The angular spacing in heliac geometry corresponds approximately to equiangular spacing in cylindrical geometry, where the areas of the sectors subtended in each case are equal. The corresponding cylindrical radius of a flux surface gives the same area as that enclosed by the flux surface. The loci of the constant azimuthal angle along the axial direction of a cylinder are assumed to follow the magnetic field lines.

### III. EXPERIMENTAL ARRANGEMENT

SHEILA is a prototype toroidal heliac with major radius 0.19 m and average minor radius 0.035 m. The magnetic field could be varied in the range 0.03-0.15 T with a pulse length of 20 ms. A schematic diagram of the apparatus and the locations of the diagnostic ports used in the experiment are shown in Fig. 4. Plasmas were produced in argon using a double saddle loop antenna and about 2.5 kW of radio frequency power at 7 MHz. A study has already been performed of the physical mechanisms responsible for plasma formation. The filling pressure was set to 2 mTorr, except in a series of experiments to be described, where it was varied to examine the effects of the filling pressure on wave coupling.
The probe leads were connected to a hybrid combiner via a of the forward and reflected RF line voltage and antenna current. Because of the frequencies involved, account had to the antenna. The effect of varying the magnetic field the antenna was located at the reentrant plasma. These could be inserted in an axial array of reentrant Pyrex sheaths, as shown in Fig. 3. A three-component magnetic probe consisting of five turns and of overall dimension 3 mm was inserted in the 0° port to measure the three components \(B_x, B_y, B_z\). The total cross-talk between each component due to nonorthogonality amounted to 7% at 18 MHz. A \(B_x\) probe consisting of ten turns and overall dimension 3 mm was mounted in a two-dimensional manipulator to perform a cross-sectional scan of the wave field. Any of these probes could be inserted in the reentrant tubes for wavelength measurements.

Because the plasma was formed by a 2.5 kW source at 7 MHz, several signal preconditioning circuits were necessary. The probe leads were connected to a hybrid combiner via a twisted pair balanced line. The hybrid combiner produces one output equal to the sum of the signals on the twisted pair and one equal to the difference, so that the user could regularly monitor the strength of the electrostatic pickup relative to the desired magnetic signal. A notch filter eliminated the 7 MHz signal. The preconditioned signal was detected by a broadband multichannel quadrature detector having a 10 kHz output bandwidth.

The antenna loading was calculated from measurements of the forward and reflected RF line voltage and antenna current. Because of the frequencies involved, account had to be taken of the length (0.6 m) of the transmission line connecting the current transformer to the antenna in estimating the antenna current. The antenna loading measurement was then checked by comparing the predicted loading with that produced by connecting various resistors across the antenna.

IV. EXPERIMENTAL RESULTS

A. Wave field measurements

The low-power helicon wave was excited at 18 MHz and the probe was located at the \(\phi=0^\circ\) location; 28.6 cm from the antenna. The effect of varying the magnetic field \(B_0\) on the wave fields is shown in Fig. 4. From Fig. 4(d), the plasma is produced and controlled by the double loop antenna, in such a way that the density increase in proportion to \(B_0\) for \(B_0>0.07\) T. To a good approximation in these experiments, the perpendicular wave number \(k\) is a constant when the parallel wave vector \(k=\beta\), so that \(k\) is proportional to the ratio \(n_i/B_0\). For our conditions, the parallel wavelength \([\text{in Fig. 4(c)]}\) and hence the phase \([\text{in Fig. 4(b)\} should therefore tend to a constant at a fixed frequency, with increasing \(B_0\).

The poloidal mode numbers were determined independently by radial profiles of the wave fields and wave field measurements along a poloidal contour close to the last closed flux surface. In Fig. 5 is shown the phase of \(B_z\) as a function of the poloidal angle in equivalent cylindrical coordinates. We conclude that the main contribution to the wave fields (at least in the peripheral region) is by \(m=+1\). Care should be exercised when comparing a poloidal profile with theory because of the radial evanescence of the high poloidal modes. For reference, we show the theoretically predicted power spectrum at 18 MHz using the model presented in Sec. II. Figure 6 plots the calculated radiation resistances for various poloidal and radial mode numbers for a plasma density \(8.0\times10^{18}\) m\(^{-3}\) on axis and \(B_0=0.14\) T. Here we assume that the effective average density is given by 0.85\(n_i(0)\). In qualitative agreement with the experiment, theory also predicts a global dominance of \(m=+1\) with a non-negligible contribution from \(m=0\). The dominance of \(m=+1\) compared with \(m=-1\) has been confirmed for a variety of plasma geometries in numerous fast and helicon wave excitation experiments.}

Radial profiles in equivalent cylindrical coordinates of the amplitude of the wave fields are shown in Fig. 7(a). These results were taken at 0.15 T, but they are not strongly affected by the magnetic field in the high-density constant wave number regime of Fig. 4. Though these profiles consist mainly of \(m=+1\) and \(m=0\), there is some evidence of \(m=-1\) due to the fact that \(B_y\) and \(B_x\) are not equal on axis. Figure 7(b) shows the theoretically predicted radial profiles.
for the first radial eigenmodes. The main contributions to this profile are indeed from \( m = +1 \) and \( m = 0 \). A small contribution from \( m = -1 \) is also evident in agreement with experiment.

### B. Wave dispersion

The dispersion relation, \( k \) versus frequency, was verified by measuring the phase of the wave fields at the 0° and 15° toroidal locations with respect to the antenna current. We are able to obtain the propagation speed parallel to \( B_0 \) and the phase with respect to the source. For these results, the magnetic field was 0.14 T and the central density, \( n_c(0) = 8 \times 10^{18} \text{ m}^{-3} \) on axis. The best fit to the experimental data shown in Fig. 8 was obtained by assuming that the effective average density used in the model be \( 0.85 n_c(0) \). Because these measurements were made using \( B_r \) on axis, we have eliminated the contribution to \( m = 0 \) in the results.

### C. Antenna radiation resistance

In ICRH, an array of phased partial-turn loop antenna elements may be used instead of a single element antenna. The wave fields launched by such an array are obtained by superposition of those for a single element antenna. In this section we aim to investigate the wave-coupling properties of the monopole half-turn loop antenna in heliac geometry. As noted previously in the wave field measurements, the dominant contribution to the radiated power is from the \( m = +1 \) first radial mode, and there is negligible power radiated into the first radial mode of the \( m = -1 \) wave compared with the first radial \( m = 0 \) and \( m = +1 \) modes. The following results may also support this argument.

Measurements were made of the antenna radiation resistance as a function of frequency for a plasma density \( 8 \times 10^{18} \text{ m}^{-3} \) and \( B_0 = 0.14 \text{ T} \). The results are the points plotted in Fig. 9. Clearly there is good agreement in both the magnitude and the frequency dependence of the antenna radiation resistance with the calculated sum of the \( m = 0 \) and \( m = +1 \) first radial mode resistances.

From Fig. 7 it appears as though the higher radial modes are not important in the experiment. It may be, however, that these modes are launched by the antenna, but are damped before reaching the probe. The total antenna loading for all poloidal and radial modes is shown in Fig. 9. The disparity between this and the experimental result further supports the conclusion that the higher radial modes are, in fact, not excited. No further investigation of the theory has been undertaken to understand this result, but a proper treatment of the plasma boundary region and the radial density profile is now under study.

Measurement of the magnitude of the wave magnetic field as a function of frequency at a fixed toroidal location were also made. Although the general magnitude of the field

![FIG. 4. The effect of varying the magnetic field \( B_0 \) on the wave fields. (a) (b) show the measured magnitudes and phases of the wave fields on axis, respectively. (c) plots the measured wavelength. (d) The plasma density is proportional to the magnetic field \( B_0 \).](image)

![FIG. 5. Poloidal profiles of the phase of the \( B_z \) field component at the 120° port. The solid line is the theoretically predicted phase and dotted line for purely \( m = +1 \) mode. Squares are the measured results. The inset indicates the magnetic flux surface.](image)

![FIG. 6. The antenna radiation resistance \( R_{rad} \) as a function of poloidal wave number for the first and second radial modes. In the calculation, a density \( n_c = 8 \times 10^{18} \text{ m}^{-3} \) with \( f = 18 \text{ MHz} \) and \( B_0 = 0.14 \text{ T} \) are assumed.](image)
agreed with theory, the frequency dependence could not be predicted due to the frequency effects on wave damping.

In Fig. 10 is shown a comparison of the experimental and calculated radiation resistance at 18 MHz as a function of the density. The density was varied in two ways by either changing the RF power in the high-power antenna responsible for plasma formation or by varying the filling pressure. These results show that the method of plasma production makes little difference qualitatively to the measured antenna loading.

Shielding the antenna to prevent coupling to other electrostatic waves or direct antenna scrape-off layer interface losses appears at least qualitatively to have served its purpose in this experiment. Direct evidence, however, for parasitic loading was not sought.

V. SCALING TO H-I

In SHEILA it is not possible to study wave propagation in hydrogen, because the available RF power would only permit densities of a few by \(10^{17}\) m\(^{-3}\). The helicon wave dispersion, however, does not depend on the ion mass, so that a scaling to H-I for plasma formation using the present theory ought to be possible, provided that \(\omega > 4\omega_{ci}\). For ICRF (ion cyclotron range heating), \(\omega > 2\omega_{ci}\), so that to estimate the antenna radiation resistance we have to include the leading terms in the dielectric tensor of Eq. (1), which properly take into account the effects of finite ion cyclotron frequency. In what follows, we assume that the antenna is a half-turn loop antenna with poloidal strap radii \(a_1=0.25\), \(a_2=0.27\), and width \(\Delta = 0.04\) m located in a cylindrical waveguide of wall radius \(b = 0.30\) m. Two sets of conditions have been considered for H-I, one typical of a plasma formation experiment and the other typical of ICRH.

The dispersion relation for the case of a plasma formation experiment with \(n_e = 8 \times 10^{18}\) m\(^{-3}\), argon filling gas, \(B_0 = 0.22\) T typical of the H-I continuous operation mode is shown in Fig. 11(a). Two sets of curves are shown for the poloidal mode numbers \(m = -1, 0\) and \(+1\) first radial modes, one for the simple helicon model and the other including finite ion cyclotron frequency. Little difference is evident between the two models, so that the results are also independent of the ion mass for plasma formation in a heavy ion gas. The resultant loading curve shown in Fig. 11(b) indicates

FIG. 7. The radial structure of the wave fields at the \(\theta^0\) port. (a) The measured profiles and (b) the theoretical predicted profiles.

FIG. 8. Verification of the dispersion relation by measuring the phase of the wave fields at the \(\theta^0 (\bigcirc)\) and \(15^0 (\star)\) with respect to the antenna current. The solid lines are the theoretical best fit to the data.

FIG. 9. The antenna radiation resistance \(R_{rad}\) (or antenna loading) as a function of RF frequency.
that a significant antenna radiation resistance is also to be expected in H-I, and that the dominant mode is now $m=0$. In practice, however, the fast wave antenna turns out to be an inefficient coupler for helicon waves compared to antennas like the double saddle coil and Nagoya type III antennas, which have $z$-directed straps. Experimental investigation of plasma formation with a fast wave antenna in SHEILA also proved difficult in comparison to plasma formation using a double saddle coil.

The dispersion relation for the case of ICRH with $n_e(0)=8 \times 10^{18}$ m$^{-3}$, deuterium main species, $B_0=1$ T, corresponding to the H-I pulsed mode is shown in Fig. 12(a). In this case, there is a noticeable difference in the two models in the low-frequency regime. The presence of the fast wave cutoff is indicative of the inclusion of finite ion cyclotron frequency effects in the model. Figure 12(b) indicates that a total loading of $\sim500$ mΩ for minority ICRH is to be expected. This value can be increased by increasing the radiating area of the antenna. This calculation is an over simplification, however, if mode conversion is involved in the wave damping process and is flawed if the damping is so low that toroidal eigenmodes can form.

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APPENDIX: ANTENNA–WAVE COUPLING THEORY

In this section we derive a theory of helicon wave excitation for the geometry of Fig. 1. We neglect all finite electron mass effects, so that $e_{\ell}=\infty$ ($E_z=0$) and $f_{\varphi}=0$. The equation of wave motion may be expressed as

$$\nabla \times \nabla \times E = i \omega \mu_0 J_\varphi + k_0^2 \varphi \cdot E,$$

where $J_\varphi$ is an external current used to model the antenna. After a Fourier transform in $z$ to the parallel wave number $k$, and taking a Fourier series in the azimuthal angle $\theta$, it is a simple matter to derive the following differential equation for $B_z$:

$$\frac{\partial^2 B_z}{\partial z^2} + \frac{1}{r} \frac{\partial B_z}{\partial r} + \left( k_z^2 - \frac{m^2}{r^2} \right) B_z = -\mu_0 (\nabla \times J_\varphi)_z,$$

where $J_\varphi=0$ has been assumed for the partial-turn loop antenna. In Fig. 1, the partial-turn loop antenna of subtended angle $\theta_A$ and axial extent $\Delta$ is immersed in a uniform magnetized plasma filled wave guide of infinite length in the $z$ direction. We let the azimuthal elements of the antenna current density be described by the following expression:

$$J_{\varphi \theta} = \frac{I_{\text{ant}}}{\Delta} \left[ U\left( \theta, -\frac{\theta_A}{2}, \frac{\theta_A}{2} \right) \right] U\left( z, -\frac{\Delta}{2}, \frac{\Delta}{2} \right) \times \left[ \delta(r-a_2) - \delta(r-a_1) \right].$$

The $\delta$ functions are the Dirac delta functions and the function $U(x,x_1,x_2)$ is equal to unity for $x_1<x<x_2$, and zero otherwise. After Fourier transforming this antenna current is represented by the following expressions for the azimuthal and radial elements:

FIG. 10. Measurements of the radiation resistance versus the plasma density $n_e$. The density variation is obtained by either the RF power scan (square) or filling gas pressure scan (triangle). A line shows the sum of the calculated radiation resistance for $m=0$, $\pm 1$ first radial modes.

FIG. 11. (a) The dispersion relation, $k_1$ versus frequency, for the H-I argon plasma with the parameters $B_0=0.22$ T, $n_e=8 \times 10^{18}$ m$^{-3}$, and major radius $R=1$ m. (b) The resultant antenna loading curves for the conditions of Fig. 10(a).

In Eqs. (A4), the radial component of the current distribution has been obtained naturally by requiring that \( \text{div}(j_r) = 0 \).

Equation (A2) can be solved by the method of the Green's functions to yield expressions for the Fourier transform for \( b_z \) in the three regions, \( r < a_1 \), \( a_1 < r < a_2 \), and \( a_2 < r \). The \( B_r \) and \( B_\theta \) components are related to \( b_z \) through the following expressions:

\[
T^2 B_r = \frac{im \alpha}{r} B_z + ik \frac{\partial B_z}{\partial r},
\]

\[
T^2 B_\theta = -\frac{m k}{r} B_z + \alpha \frac{\partial B_z}{\partial r}.
\]  

The electric fields are obtained from Faraday's law,

\[
E_r = \frac{\omega}{k} B_\theta,
\]

\[
E_\theta = -\frac{\omega}{k} B_r.
\]  

The radiation resistance for a given poloidal mode number can be obtained from the following expression:

\[
R_{rad} = \frac{1}{P_{ant}} \int_0^{2\pi} \int_0^\pi \int_0^\infty r \, dr \, \frac{dk}{2\pi} \text{Re}(E_r J^*_r + E_\theta J^*_\theta). \tag{A7}
\]

If the Fourier transform with respect to \( k \) is inverted using the residue theorem, the following expression is obtained:

\[
R_{rad} = \frac{4 \mu_0 \omega H F J \sin^2(m \theta_A/2) \sin^2(k \alpha \Delta/2)}{k^2 T^2 \Delta^2 m^2 (\partial D/\partial k)^2}.
\]  

\[ \tag{A8} \]

where

\[
F = \frac{m \alpha_j}{b} Y_m(T,b) + k_j T_m b,
\]

\[
H = \frac{m \alpha_j}{b} J_m(T,b),
\]

\[
D = \frac{m \alpha_j}{b} J_m(T,b) + k_j T_m b
\]

\[ J = m \alpha_l [J_m(T,a_2) - J_m(T,a_1)] \]

\[ + k_j T_m [a_2 J^*_m(T,a_2) - a_1 J^*_m(T,a_1)] \]

\[ - m \int_{a_1}^{a_2} \frac{dk_j}{r} J_m(T, r) + \alpha_l T_m(J^*_m(T,r)) \]

and the subscript \( l \) refers to the solutions to \( D = 0 \) corresponding to the different radial modes.

Equation (A8) can also be obtained from Poynting's theorem for zero damping as follows:

\[
R_{rad} = \frac{1}{\mu_0^2 P_{ant}} \int_0^{2\pi} \int_0^\pi \int_0^\infty r \, dr \, \frac{dk}{2\pi} \text{Re}(E_r B^*_r - E_r B^*_r).
\]  

\[ \tag{A9} \]

In order to evaluate expression (A9), we first invert the Fourier transforms of the fields components with respect to \( k \), and once again retain only those terms obtained from the residue theorem. The resulting expression is given by

\[
R_{rad} = \frac{4 \pi \mu_0 H F J \sin^2(m \theta_A/2) \sin^2(k \alpha \Delta/2)}{k^2 T^2 \Delta^2 m^2 (\partial D/\partial k)^2}.
\]  

\[ \tag{A10} \]

where

\[
I = \int_0^b r \, dr \left( a^2 + k^2 \left( \frac{m^2}{r^2} J_m^2(T(r)) + \frac{2 \alpha^2}{m} J^*_m(T(r)) \right) \right).
\]

Expressions (A8) and (A10) give identical results and have been used in this paper for comparison with experiment.


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