Snapshot-imaging motional Stark effect polarimetry

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Abstract

Measurement of the motional Stark effect (MSE) for Balmer alpha light emitted from heating or diagnostic neutral beams is a standard technique for estimating plasma toroidal current density in tokamaks. Most techniques typically rely on a determination of the polarization angle or relative intensities of the multiplet components which are spectrally resolved using either an array of interference filters or a high-throughput grating instrument. Neither of these approaches is amenable to two-dimensional MSE imaging. This paper proposes an alternative measurement scheme that is suitable for single snapshot two-dimensional imaging of the current distribution. This is achieved using a spectro-polarimeter that encodes the polarization and spectral information upon spatially orthogonal linear interference fringe patterns illuminated by the image of the Stark-shifted H-alpha emission from the neutral beam. The new technique opens the possibility to use synchronous detection methods and gated cameras to resolve the magnetic structure of periodic tokamak phenomena including sawteeth and magnetohydrodynamic activity.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Motional Stark effect (MSE) polarimetry is now a standard method for estimating magnetic field pitch angle in tokamaks equipped with high power heating or diagnostic neutral beams [1–4]. However, given the technical difficulties, especially in low field compact systems where the Stark multiplet is difficult to resolve or is contaminated by other spectral features, there has been no consideration given to imaging possibilities. In this paper I propose an optical system that can capture the full polarimetric information about the entire Stark multiplet in a single snapshot. This is achieved by using interfero-polarimetric methods to produce orthogonal spatial carrier fringes which encode the polarimetric information. This information is insensitive to arbitrary unpolarized spectral contamination such as the wing of the Hα emission or leakage from adjacent beam energy components. Under certain conditions, it is also insensitive to background radiation that has become polarized due to reflections from various surfaces.
The MSE technique relies on the splitting of the Doppler-shifted neutral beam Balmer $\alpha$ light into orthogonally polarized $\sigma$ and $\pi$ components as a result of the motion-induced strong electric field $E = v \times B$ experienced in the rest frame of the neutral atoms. When viewed in a direction perpendicular to $E$ the Stark split $\sigma$ and $\pi$ components are polarized, respectively, perpendicular and parallel to the direction of $E$. When viewed along $E$ the $\sigma$ components are unpolarized and the $\pi$ components have no brightness. The Stark separation of adjacent Balmer alpha spectral components varies as $\Delta\lambda_S = 2.7574 \times 10^{-8}E$ nm where $E = |v \times B|$ is the induced electric field [5]. Integrated over wavelength, the Stark multiplet is nett unpolarized and no orientational information can be obtained.

The magnetic field pitch angle is usually estimated by isolating and measuring the polarization direction of the central cluster of $\sigma$ lines. This requires a tunable narrowband filter to spectrally resolve the multiplet in order to obtain a nett polarization that is analyzed by a modulated polarimeter. Every spatial channel requires a dedicated filter whose passband must be optimized by tilt or thermal tuning. The filters are lossy and when the splitting is small, the nett polarization can be low. The requirement to adequately resolve the $\sigma$ and $\pi$ clusters strongly constrains the range of beam energies, divergences and energy spreads and magnetic fields for which the method is useful [5, 6]. The use of very narrowband filters also complicates the interpretation of beam into gas calibration procedures [7].

In an earlier paper [8], I proposed an alternative MSE measurement scheme that combined a modulated polarimeter and fixed delay polarization interferometer to monitor the temporal coherence of light from the full Stark multiplet. The polarization discriminator modulates the spectral content of the light which is then sensed by the interferometer as a variation in the optical coherence. Because the narrowband filter is not required, the system is not susceptible to many of the above-noted problems, whilst also obtaining higher light throughput and near-optimum polarization contrast.

While sharing aspects of this approach, the scheme demonstrated here captures all of the polarization information in a single image by using spatial heterodyne techniques to encode the polarization orientation on orthogonally oriented fringes. The compromise is that spatial resolution is limited roughly to the wavelength of the imposed fringe patterns and temporal resolution is constrained by the image frame rate. Nevertheless, because of the spatial encoding and the ability to accept a relatively wide spectral window, it becomes feasible to undertake two-dimensional current profile imaging. Periodic phenomena such as reconnection events and magnetohydrodynamic activity can be resolved temporally using synchronous gated intensified cameras. Also note that when traditional single channel measurements are required, the proposed optical system reduces to a particularly simple arrangement comprising a wideband interference filter, a single polarization modulator, a birefringent delay plate and analyzer.

This paper is organized as follows. Section 2 gives a broad description of the optical system and its operating principle. A mathematical analysis which quantifies the instrument response is then developed in section 2.3, while in section 3 I present a simple and efficient single channel spectro-polarimeter applicable in situations where imaging techniques are not practical. To illustrate how the imaging system might perform in a tokamak environment, I have modeled the performance under conditions appropriate for MSE measurements on the TEXTOR tokamak [9] (section 4). This modeling was undertaken at the Australian National University in support of an upcoming collaboration between the FOM Institute for Plasma Physics and the ANU on the development of a multi-channel MSE system for TEXTOR. In section 5 the system imaging performance is confirmed by the observation of a simple Zeeman triplet produced by a magnetized zinc discharge lamp. Finally, section 6 considers the various practical issues, including polarization self-calibration, which help optimize the design of an imaging MSE system.
2. Imaging system for MSE polarimetry

2.1. Measurement principle

In the absence of spectral discrimination, the emission from the Stark multiplet is net unpolarized. The idea behind the new method is to simultaneously observe both $\sigma$ and $\pi$ clusters of lines using an interferometer (wave plate) to obtain the necessary spectral discrimination and a polarizer to extract the polarization orientation. Under quasi-monochromatic illumination at optical frequency $\nu_0$, a simple polarization interferometer consisting of polarizer, birefringent optical delay plate and analyzer produces an output signal

$$S_{\pm} = \frac{I}{2}(1 \pm \zeta \cos \phi),$$

where $I$ is the irradiance, $\phi = 2\pi \nu_0 \tau$ is the phase delay corresponding to time delay $\tau$ and $\zeta(\tau)$ is the fringe visibility or contrast at time delay $\tau$. The plus and minus signs depend on whether the first polarizer is parallel or perpendicular to the final analyzer.

Consider illumination that is already linearly polarized and oriented at some angle $\theta$ with respect to the analyzer. Dispensing with the first polarizing element, the nett signal is the sum of interferograms corresponding to the incident electric field vector components parallel and perpendicular to the final analyzer:

$$S = \frac{I}{2} \cos^2 \theta (1 + \zeta \cos \phi) + \frac{I}{2} \sin^2 \theta (1 - \zeta \cos \phi) = \frac{I}{2} (1 + \zeta \cos 2\theta \cos \phi).$$

The effective fringe visibility is therefore proportional to the polarization orientation. If the interferometer delay is zero (remove delay plate) we have the familiar result for transmission through a simple polarizer.

When illuminated by the MSE multiplet (ignoring unpolarized components), the resulting signal can be written as

$$S = \frac{I_\pi}{2} (1 + \zeta_\pi \cos 2\theta \cos \phi_\pi) + \frac{I_\sigma}{2} (1 - \zeta_\sigma \cos 2\theta \cos \phi_\sigma),$$

where the minus sign arises as a result of the orthogonality of the component clusters. Assuming, for simplicity, equal component irradiances $I_\pi = I_\sigma = I_0$, and identical center-of-mass wavelengths ($\phi_\pi = \phi_\sigma$), equation (3) reduces to

$$S = I_0 (1 + \zeta \cos 2\theta \cos \phi)$$

where $\zeta = \zeta_\pi - \zeta_\sigma$. With an appropriate choice of optical delay $\tau$, it is possible to obtain a nett non-zero effective fringe contrast $\zeta(\tau)$ which carries the angle of inclination of the multiplet polarization.

In the following sections, we expand on this idea and show how the polarization angle can be encoded as a phase modulation of temporal or spatial heterodyne carriers. Because the new approach relieves the constraint of having to spectrally isolate the $\sigma$ cluster of lines, it also allows the possibility of 2D imaging of the heating beam using a wide bandpass interference filter.

2.2. Optical system description

We take light to be propagating in the $z$-direction, parallel to the axis of the optical system. We assume that the full multiplet can be isolated from other spectral features using a suitably chosen interference filter (though this condition is relaxed below). To obtain polarization information (and hence the magnetic field pitch angle) it is necessary to apply some spectral discrimination that resolves the multiplet. For this purpose we use polarization interferometric techniques
Figure 1. Top: optical arrangement for producing spatial heterodyne images of the spectrally resolved polarization state for 2D imaging of the Stark multiplet in neutral beam heated plasmas. Bottom: showing the polarization states for the wave field components as they traverse the instrument. The x and y lateral shears give rise to a two-dimensional interference pattern in the lens focal plane.

to generate optical fringes along the x-axis of the 2D detector array which encode the optical coherence of the imaged scene about some fixed optical delay offset. One can consider these fringes as being produced by a sinusoidal spectral filter which shifts across the multiplet to give rise to regions of alternating high and low relative brightness. The fringe contrast (visibility) and fringe phase depend on the wavelength of the light and the filter period and phase (set by the optical path delay). In a similar fashion, a set of interference fringes imposed along the y-axis encodes the orientation of the multiplet. For a linearly polarized scene, the visibility of y-fringes formed at the focal plane of a lens varies as the cosine of twice the angle between the incident polarization vector and the axis of the final analyzer (see equation (4)). When both orthogonal Stark components are present, the polarization orientation is revealed provided there is some appropriate spectral discrimination.

The first element in the optical chain (see figure 1) is a zero-nett-delay double Savart prism (see appendix A). The plate is oriented so that its principal section is at −45° to the system x-axis, thereby giving rise to virtual sources separated in the y-direction and polarized at ±45°. Since we will be discussing the temporal coherence of the multiplet, it is convenient to express the OPD for the components in the optical system in terms of an equivalent time delay $\tau = \Delta /c$. We denote the orthogonal components emerging from the double Savart plate by $E_1(\tau_1)$ and $E_2$ where $\tau_1(y)$ represents the focal-plane time-delay shear in the y-direction.

A wave plate having fast axis in the x-direction follows the Savart plate. This ‘primary’ wave plate introduces a mutual delay $\tau_2$ between the orthogonal components of each of the waves $E_1$ and $E_2$. The delay is chosen to be comparable to the temporal coherence of the multiplet, thereby providing a degree of spectral discrimination that allows the polarization state to be resolved. This delay (in waves) is typically of the order of the inverse of the
normalized spectral width $N \sim \lambda_0 / \Delta \lambda$ and gives rise to a non-zero effective nett contrast

$$\zeta = \zeta_0 - \zeta_\sigma.$$

The primary wave plate is followed by a single plate Savart prism with optic axis in the $x$–$z$ plane (co-aligned with the primary wave-plate). This prism separates the waves in the $x$-direction, introducing an additional small fixed delay offset and a focal-plane $x$-dependent time-delay shear. Together, the primary wave plate–Savart plate combination brings about an $x$-dependent delay $t_2(x)$ comprising a fixed offset and a linear shear. A final analyzer oriented at 45°, and lens cause the various wave $x$ and $y$ polarized wave fields to interfere to produce orthogonal vertical and horizontal fringe patterns on the detector array. The path difference at position $x$ from the origin in the focal plane depends on the Savart plate thickness (beam separation $d$) and the lens focal length according to (see equation (30)) $\Delta = dx / f$.

A polarizing Wollaston prism could also be used as the final analyzer. In this case, dual anti-phase interferometric images are formed on the CCD. The approach has the advantage that no light is lost at the final analyzer. However, the image needs to be corrected for optical aberrations and the dual images spatially registered in order to take advantage of the additional light. If the system etendue is not limited by the imaging optics, a comparable fluence is obtained by simply forming a larger image on the detector array.

2.3. Analysis of the optical system response

Using a Jones matrix analysis, the irradiance in the focal plane can be written in terms of the temporal average of the squared sum of the four relatively delayed wave components:

$$I = \langle |(E_1(t + \tau_1) + E_1(t + \tau_1 + \tau_2)] + [E_2(t) - E_2(t + \tau_2)]|^2 \rangle.$$ (5)

The negative sign in the second term arises because of the orthogonal polarization of the rays produced by the first shear plate. Within a constant factor, equation (5) yields

$$I(x, y) = I_1 - \Gamma_{11}(\tau_2) + I_2 + \Gamma_{22}(\tau_2) + \Gamma_{12}(\tau_1 + \tau_2) - \Gamma_{12}(-\tau_1 + \tau_2),$$ (6)

where the real part of the coherence between the waves $E_1(t)$ and $E_2(t + \tau)$ at time delay $\tau$ is given by [10]

$$\Gamma_{12}(\tau) = \Re(\langle E_1(t)E_2^*(t + \tau) \rangle)$$ (7)

and $I_1 = \Gamma_{11}(0)$ and $I_2 = \Gamma_{22}(0)$. As expected, when the birefringent elements are removed ($\tau_1 = 0$, $\tau_2 = 0$), equation (6) gives $I = I_2$ corresponding to the analyzer being oriented to transmit only the electric field component $E_2$.

In order to simplify further we need to consider the temporal coherence (spectral content) of the orthogonal wave fields $E_1$ and $E_2$. The amplitudes of these components depend on the orientation of the Stark multiplet with respect to the axes of the imaging system. We consider the viewing geometry shown in figure 2 where $\beta$ is the angle between the $x$-axis and the projection onto the $x$–$y$ plane of the induced local Stark electric field and $\gamma$ is the polar angle with respect to the imaging system.

It can be shown that the Stokes vectors for the orthogonally polarized components of a Zeeman multiplet can be expressed in terms of the orientation angles ($\beta, \gamma$) as [11]

$$s_\sigma = I_\sigma / 2$$ (8)

$$s_\pm = I_\pm (1 + \cos^2 \gamma, -s_1, -s_2, \mp s_3),$$ (9)

$$s_\mp = I_\mp = I_\sigma / 2$$ are the component irradiances,

$$\langle s_1, s_2, s_3 \rangle = (\sin^2 \gamma \cos 2\beta, \sin^2 \gamma \sin 2\beta, 2 \cos \gamma).$$ (10)
and \( s_1^2 + s_2^2 + s_3^2 = s_0^2 \). The situation is somewhat different for the MSE multiplet in that the \( \sigma_+ \) and \( \sigma_- \) transitions are superimposed, leading to \( s_3 = 0 \), and therefore partially polarized \( \sigma \) radiation for which \( s_1^2 + s_2^2 + s_3^2 < s_0^2 \). The polarized component irradiances \( I_\sigma \) and \( I_\pi \) (summing + and − \( \pi \) clusters) remain equal, both scaling in brightness according to the factor \( \sin^2 \gamma \). The polarized fraction \( (1 - \cos^2 \gamma)/(1 + \cos^2 \gamma) \) of the total \( \sigma \) radiation is a maximum when viewing in a direction orthogonal to the induced electric field. In the following discussion, we disregard the effects of the unpolarized components due to polar angle \( \gamma \neq \pi/2 \) and also ignore the common \( \sin^2 \gamma \) inclination factor. These unpolarized components have the effect of reducing the polarization contrast and are of no consequence for the immediate analysis. However, all components are retained when modeling the nett contrast (section 4).

Given the \(-45^\circ\) orientation of the first Savart plate, the components \( E_1 \) and \( E_2 \) can be written in terms of the angle \( \theta = \beta + \pi/4 \) referred to the Savart plate axes and the radiated fields \( E_\pi \) and \( E_\sigma \) as

\[
E_1 = E_\pi \cos \theta - E_\sigma \sin \theta, \\
E_2 = E_\pi \sin \theta + E_\sigma \cos \theta, 
\]

where \( E_\pi \) includes contributions from both shifted \( \pi \) manifolds. Because the emission from distinct transitions within the multiplet is uncorrelated, the cross correlations vanish. Thus, substituting from equation (11) into equation (6) gives

\[
I(x, y) = I_\pi + I_\sigma + [\Gamma_\sigma(\tau_2) - \Gamma_\pi(\tau_2)] \cos 2\theta + \frac{1}{2} [\Gamma_\sigma(-\tau_1 + \tau_2) - \Gamma_\sigma(\tau_1 + \tau_2) \\
- \Gamma_\pi(-\tau_1 + \tau_2) + \Gamma_\pi(\tau_1 + \tau_2)] \sin 2\theta, 
\]

where the subscripts \( \pi \) and \( \sigma \) refer to their respective clusters of spectral lines. In general, the complex temporal coherence is related to the spectral irradiance \( I(\nu) \) through the Wiener–Khinchine theorem

\[
\Gamma(\tau) = \int_{-\infty}^{\infty} I(\nu) \exp(2\pi i \nu \tau) \, d\nu. 
\]

For quasi-monochromatic light the self-coherence of a spectral feature can be written in terms of its fringe visibility \( \zeta = |\Gamma(\tau)| \) and phase as

\[
\Gamma(\tau) = I(\nu_0) \cos[2\pi \nu_0 \tau + \alpha(\tau)], 
\]
where $I$ is the irradiance, $\alpha(\tau)$ is a slowly varying phase which vanishes when the line-shape is symmetric and $v_0$ is the line center frequency. For polarization interferometers, the optical path delay $\tau = L B(v)/c$ depends on the optical-frequency-dependent wave-plate birefringence $B(v)$ and its thickness $L$. When the medium is dispersive, the contrast $\zeta$ and phase factor $\alpha$ in equation (14) become functions of the group delay $\hat{\tau} = \kappa \tau$ [11], where

$$\kappa = 1 + \frac{v_0}{\tau_0} \frac{\partial \tau}{\partial v} |_{v_0}. \tag{15}$$

Hereafter we assume this effect to be understood.

In principle, the $\pi$ and $\sigma$ manifolds have the same effective central wavelength, and so the same interferometric phase $2\pi v_0 \tau$. However, allowing for possible asymmetries in the optical system transmission passband or line-shape we write $\phi_\pi(\tau) = 2\pi v_0 \tau + \alpha_\pi$ and $\phi_\sigma(\tau) = 2\pi v_0 \tau + \alpha_\sigma$. Substituting from equation (14) into equation (12) and simplifying then gives an expression for the irradiance distribution in the focal plane

$$I(x, y) = I_\pi + I_\sigma + [I_\sigma \zeta_\sigma - I_\pi \zeta_\pi] \zeta_A(\tau) \cos 2\theta \cos \phi_2(x)$$

$$+ [I_\sigma \zeta_\sigma - I_\pi \zeta_\pi] \zeta_A(\tau) \sin 2\theta \sin \phi_2(x) \sin \phi_1(y), \tag{16}$$

where, because the polarimeter phase delay $\tau_1$ introduced by the Savart plate is small compared with the interferometric delay $\tau_2$, we have taken $\zeta(\pm \tau_2 + \tau_1) \approx \zeta(\tau_2)$. The spatial heterodyne carrier wave phases are $\phi_2(x) = 2\pi v_0 \tau_2(x) + \alpha(x)$ and $\phi_1(y) = 2\pi v_0 \tau_1(y)$. The contrast change due to possible spectral asymmetries is given by $\zeta_A(\tau) = \sqrt{\sin^2 \alpha_\pi + \cos^2 \alpha_\pi}$ and its associated phase by $\alpha(\tau) = \arctan(\sin \alpha_\pi / \cos \alpha_\pi)$. For simplicity, we set $\zeta_A = 1$ and $\alpha = 0$ and examine numerically the effects of asymmetries in section 4. In practice, unless $\alpha$ varies more rapidly than a fringe period, its value is of no consequence for recovery of the polarization information. Observe that equation (16) can be written in the more instructive form

$$I(x, y) = I_0 [1 + \zeta \cos 2\theta \cos \phi_2(x) + \sin 2\theta \sin \phi_2(x) \sin \phi_1(y)], \tag{17}$$

$$I_0 = I_\pi + I_\sigma, \tag{18}$$

$$\zeta = \frac{I_\sigma \zeta_\sigma - I_\pi \zeta_\pi}{I_\pi + I_\sigma}, \tag{19}$$

where $I_0$ is the total multiplet irradiance and $\zeta$ is a nett fringe contrast that depends on the polarized component relative intensities (nominally $I_\pi = I_\sigma$), as well as their separation and broadening. One can interpret the interference pattern as arising from the transmission of the multiplet spectrum through a spatially periodic sinusoidal spectral filter.

The key result of the analysis is that, provided there is a measurable nett contrast $\zeta$ that does not change significantly over the spatial modulation period, a simple demodulation procedure is sufficient to recover the polarization orientation projected onto the measurement plane. For example, in the simplest approach, summing the demodulated phases of the fringe patterns along image rows where $\sin \phi_1(y) = \pm 1$ gives the quantity $4\theta$. The factor of four suggests that this differential method should be sensitive to small changes in polarization orientation $\theta$. A non-vanishing nett contrast can be obtained almost regardless of spectral details such as the relative component intensities, non-ideal transmission of the pre-filter passband, background contamination or even the presence of half and third energy components. Indeed, inclusion of these components could enhance the overall signal-to-noise ratio. This is considered further in section 4.

It is necessary to explore the conditions under which the nett contrast $\zeta$ of the spatial carriers is maximized, and to estimate the maximum contrast that can be achieved. As a first approximation, the contrast of the $\pi$ clusters is given by

$$\zeta_\pi = \zeta_0 \cos \Delta \text{ where } \zeta_0 = \zeta_{\pi+} = \zeta_{\pi-}$$
is the contrast of the upper and lower π clusters separately and Δ = 2πΔνSτ is the phase shift associated with the effective Stark separation ΔνS of the π clusters from the σ components. The cos Δ term is a result of the beating of the π+ and π− interferograms. When the broadening ΔνD of the clusters is dominated by beam energy noise or geometric effects, we can approximate ζ0 ≈ ζ and, for simplicity, take the irradiances IS and IS to be equal, to obtain ζ = ζ0(1 − cos Δ)/2. To interpret this result, recall that the contrast ζ0 decreases with increasing Doppler width ΔνD of the component clusters. As expected then, the expression for ζ shows that the optical delay τ that maximizes ζ depends on the ratio of the separation ΔνS compared with the spectral broadening ΔνD of the polarized clusters. When the broadening is small, the optimum delay occurs when Δ = π corresponding to φ2 = π(ν0/ΔνD), or N = ν0/2ΔνS where N is the number of waves delay introduced by the primary optical delay plate.

A plot of maximum nett contrast versus the ratio ΔνD/ΔνS is shown in figure 3. We have taken a Gaussian spectral line-shape exp(−(ν − ν0)2/Δν2D) for which the fringe contrast takes the form ζ0(τ) = exp[−(πΔνDτ)2]. The insets show the spectral shape of the multiplet at ratios 0.5, 1 and 2, and illustrate that reasonable contrast >0.1 can be achieved even when the components are strongly blended.

Finally, the presence of unpolarized contamination within the target passband simply adds to the total irradiance on the right side of equation (17), regardless of its spectral content, and so does not interfere with the polarization angle information. To see this, note that the intensities and contrasts of the orthogonal background components must be identical, and as a result have vanishing nett contrast (see equation (19)). Indeed, even if the background is polarized, it will likely be spectrally broad compared with the σ−π separation and therefore have negligible fringe contrast at the target optical delay. The presence of a broad, possibly polarized background is therefore to reduce the nett contrast of the spatial carriers while leaving intact the Stark multiplet polarization information.

3. Hybrid and single channel systems

Hybrid systems that employ both spatial and temporal heterodyne techniques may have advantages in some applications. Fast switching (≈10 µs) ferro-electric liquid crystal (FLC) wave plates synchronized with the imaging camera can be used in place of the first Savart plate to temporally modulate the polarization state. If the first Savart plate is removed (τ1 = 0), equation (16) becomes

\[ I = I_0(1 + \zeta \cos 2\theta \cos \phi_2) \]  

(20)
and only the spatial heterodyne carrier \( \phi_2(x) \) remains. (If the second Savart plate is also removed, the OPD \( \phi_2 \) retains only the fixed offset imposed by the primary wave plate and equation (20) is the same as equation (4)). Without polarization modulation, the orientation \( \theta \) can be recovered only when the spectral contrast \( \zeta \) is known.

Consider an FLC quarter-wave plate as the modulating element aligned such that the plate axes in the unswitched state are parallel to those of the primary wave plate. In this case the plate contributes an additional quarter wave to the path delay introduced by the primary wave plate. In the switched state, the FLC axes rotate by \(-45^\circ\) introducing an effective delay \( \phi_1 = \pi/2 \), and the images in the two states are given, respectively, by

\[
I_- = I_0[1 - \zeta \cos \phi_2 - \theta), \tag{21}
\]
\[
I_+ = I_0[1 + \zeta \cos(\phi_2 - 2\theta)] \tag{22}
\]

and correspond in turn to amplitude and phase modulated carrier fringes. A straightforward demodulation procedure should enable recovery of the polarization tilt angle \( \theta \).

It is possible to obtain a purely phase modulated outcome by combining a quarter-wave plate and half-wave FLC. If the first Savart plate is replaced by a quarter-wave plate \( \phi_1 = \pi/2 \), equation (17) becomes

\[
I = I_0[1 + \zeta \cos(\phi_2 - 2\theta)] \tag{23}
\]

and, in the imaging case, the polarization orientation is carried as a phase modulation of the spatial carrier \( \phi_2(x) \). In practice, the carrier phase will also vary due to Doppler shifts of the multiplet center wavelength and asymmetries in the spectral profile which change the effective center wavelength \( \nu_0 \), so polarization modulation methods are again required. For this purpose, the quarter-wave plate is followed by a half-wave FLC again aligned such that the plate axes in the unswitched state are parallel to those of the following primary wave plate. In the unswitched state, the FLC plate introduces an additional \( \pi \) shift in the interferometric phase offset \( \phi_2 \). In the switched state, this \( \pi \) phase shift appears in \( \phi_1 \) and the resulting image states are

\[
I_- = I_0[1 - \zeta \cos(\phi_2 - 2\theta)], \tag{24}
\]
\[
I_+ = I_0[1 + \zeta \cos(\phi_2 + 2\theta)]. \tag{25}
\]

The image pair together can be demodulated for the polarimetric phase without regard to spectral details such as the interferometric phase offset \( \phi_2 \) or the effective contrast \( \zeta \) provided they are not changing more rapidly than the camera frame rate.

A slightly modified modulation strategy is required in order to take advantage of the system merits for discrete channel 1D systems. In this case the quarter-wave and FLC plates are replaced by a single photo-elastic modulator of quarter-wave amplitude. It is clear from equation (23) that the received intensities at the modulation values \( \sin \phi_1 = \pm 1 \) and \( \sin \phi_1 = 0 \) are sufficient to deliver the polarization tilt angle. The demodulation procedure can be simplified under the assumption that the orientation \( \theta \) is small, in which case equation (23) becomes

\[
I = I_0[1 + \zeta \cos(\phi_2 - 2\theta \sin \phi_1)], \tag{26}
\]

where \( \phi_1 = (\pi/2) \sin \omega t \). If desired, a second PEM could be used to modulate the interferometric delay \( \phi_2 \), enabling the polarimetric and interferometric phases to be easily distinguished. Figure 4 shows a generic schematic layout for a simple single channel system. The instrument performance as described by equation (23) has been observed using a bench-top apparatus viewing a magnetized lamp.
4. Modeling results for MSE imaging on TEXTOR

The optical system behavior can be illustrated using model spectra calculated for MSE conditions appropriate for the TEXTOR tokamak [9]. A simple 2D code has been developed which calculates the spectra and polarization states in the case where both the beam and the optical system z-axis reside in the horizontal midplane. Specifically, the model results are used to calculate the Stokes components as a function of wavelength and observation angle for the observation geometry presented in [9]. The spectra allow calculation of the interferograms and thereby the modeling of the instrument response, including the effects of background contamination and spectral transmission asymmetries.

The neutral beam, which is injected toroidally, is modeled as an array of beamlets having spatial and angular distribution consistent with the known beam properties (H beam, \( E = 50 \text{ keV}, \Delta E/E = 0.01, 1/e \text{ beam divergence } 1.2^{\circ}, \text{ beam FWHM at port entrance } 0.2 \text{ m} \)). The toroidal field on axis (\( R = 1.75 \text{ m} \)) is 2.25 T and the toroidal current profile is parabolic with total current 350 kA. In calculating the Stokes vector for the line integrated emission in this simple model, the radiant flux from each beamlet is taken to be the same (i.e. no plasma density or temperature effects are included) and optical collection solid angle effects are ignored.

Figure 5 shows spectra, component contrasts and nett contrast profiles for full energy MSE \( \pi \) and \( \sigma \) emission from the plasma edge (tangency radius 2.10 m) and center (tangency radius 1.74 m). The spectra are asymmetric as a result of the effects of both changing Doppler shift and Stark separation across the width of the neutral beam. The fringe contrast profiles represent the modulus of the Fourier transform of the corresponding spectral components. The higher frequency beating apparent on the \( \pi \) profiles is due to the interference between the fringe patterns for the upper and lower \( \pi \) clusters. The decay length of the \( \sigma \) contrast profile is related to the width of the Doppler broadened \( \sigma \) cluster of lines.

Because the phase of the \( \pi \) component fringes changes sign about the first contrast zero-crossing, the nett contrast continues to grow in amplitude with optical delay, reaching a maximum close to the second contrast peak where \( \Delta = \pi \). Note that the observed delay of \( \sim 1000 \) waves for maximum nett contrast is not too different from the optimum condition \( N = \nu_0/2\Delta\nu_S = \lambda_0/2\Delta\lambda_S \), where \( \lambda_0 \sim 662 \text{ nm} \) and \( 2\Delta\lambda_S \sim 0.9 \text{ nm} \) (see discussion above). The reasonably high contrast of \( \sim 0.8 \) in the edge region (case (b)) is due to the relatively well resolved \( \pi \) and \( \sigma \) clusters. The maximum effective contrast reduces when the component separation is small. The reduction in nett contrast due to unpolarized \( \sigma \) radiation is negligible in these simulations.
4.1. Effects of contamination in the filter passband

One of the difficulties with many MSE polarimetric measurements is the need to use a tunable narrow filter to spectrally resolve the multiplet in order to measure the nett polarization. By admitting the full multiplet, the present method obviates many of these problems. However, in an imaging application, because of the range of beam Doppler shifts, it is necessary to employ quite a wide passband. Unpolarized background, for example from the wing of the H\textalpha line, causes a reduction in nett fringe visibility. Because this is difficult to model, we instead assess the effect of polarized contamination from half and third energy component spectra.

Contours of the peak-normalized MSE spectra and their associated nett contrast profiles versus tangent radius are presented in figure 6. We take an ideal rectangular passband filter that admits only the spectral region between 661 and 664 nm and we ignore the shift in filter passband with an angle of incidence. As seen in figure 6(a), the Doppler red shift and Stark splitting are seen to vary as expected with beam viewing angle. From figure 6(b) it is apparent that a primary wave plate of effective delay $\sim 1000$ waves at 662 nm (e.g. 3.5 mm thick calcite) will ensure good contrast for the polarimetric measurements across the wide ($25^\circ$) field of view. In practice, one might adjust the optical system in such a way that the filter passband shift with incidence angle occurs in the same direction as the multiplet Doppler shift. Furthermore, the optical delay shear $\tau_2(x)$ can be imposed in such a way as to track the shift in the peak of the nett contrast with view angle.

Assuming the various beam energy component spectra share a common polarization, it is interesting to calculate the nett contrast in the case that all spectra are observed simultaneously. This would certainly increase the collected light flux and allow the use of a much wider bandwidth optical pre-filter. The results presented in figure 7 show that, though the regions of high nett contrast are narrower (the spectrum is wider and more complex), significant nett contrast persists so that in some circumstances there may be advantages to this approach.

5. Experimental verification using a magnetized lamp

To verify the system performance we have measured the orientation of the Zeeman split $^3S_1-^3P_0$ transition at 468 nm ($\kappa = 1.60$) in a zinc dc discharge lamp (18 V, 2 A) placed between the
Figure 6. Contours of (a) spectrum and (b) nett contrast versus viewing chord angle spanning the plasma edge to beyond the magnetic axis. Note the change in Doppler shift and Stark separation with view angle. The nett contrast is a maximum near 1000 waves delay. The position of maximum delay is inversely proportional to the Stark separation. See text for discussion.

Figure 7. Contours of (a) spectrum and (b) nett contrast versus viewing chord angle spanning the plasma edge to beyond the magnetic axis. As observed in TEXTOR, the half energy spectrum is modeled as 50% brighter than full and third energy multiplets. Multiple regions of reasonable nett contrast suggest that observing all MSE manifolds may yield signal-to-noise ratio and/or time resolution improvements over the case where only the full energy components are observed. See text for discussion.

The upper and lower electronic levels of an atom or ion in a weak magnetic field are split into $2J + 1$ sublevels with energy shifts given by

$$\Delta E_{JM} = g_{\alpha JM} M \mu B,$$

where, for $L - S$ coupling, $g_{\alpha JM}$ is the Landé splitting factor, $\mu$ is the Bohr magneton and $\alpha$ denotes quantum numbers other than the total angular momentum $J$ and magnetic quantum number $M$, where $|M| \leq J$. Because of these shifts, the spectral line is split into a number of components whose relative intensities and polarizations depend on the corresponding changes in $J$ and $M$ [12, 13]. Viewed along $B$, the emission is zero for the $\Delta M = 0$ ($\pi$) lines and is circularly polarized clockwise for $\Delta M = +1$ and counterclockwise for $\Delta M = -1$ ($\sigma$ multiplets). Viewed in the direction perpendicular to $B$, the emission is linearly polarized perpendicular to $B$ for $\Delta M = \pm 1$ and parallel to $B$ for $\Delta M = 0$. The polarization properties are quite similar to those for an MSE multiplet.
Figure 8. Images of magnetized zinc lamp at 468 nm. The 2D fringe pattern is a result of combined spectral (x-direction) and polarimetric (y-direction) spatial heterodyne interferometry. The magnetic field is oriented in the plane of the image. Left to right and top to bottom: the polarization of the multiplet is rotated in 10° increments using a half-wave plate. The numbers indicate the nominal values for the polarization angle $\theta$. The image size is 260 $\times$ 220 pixels.

The light is observed in a direction perpendicular to the magnetic field $\gamma = \pi/2$. The first double Savart prism ($\tau_1$) uses two crossed calcite plates each of thickness 0.5 mm to generate fringes with wave-vector $k_y$ in the vertical $y$ direction. A single calcite Savart plate of thickness 1.5 mm is used as the final shearing element to produce fringes with wave-vector $k_x$ in the $x$ direction. As above, for optimum fringe contrast it is necessary that the magnetic field dependent difference-phase $\Delta$ be of order $\pi$ radians. Because the maximum field strength achievable is of order 0.4 T, the Zeeman splitting is small and field-widened lithium niobate wave plates of total thickness 90 mm are required to obtain good nett contrast ($N \sim 30,000$) [11]. A half-wave plate interposed between the lamp and the optical setup is used to artificially rotate the multiplet polarization about the optical system $z$-axis in order to test the capability of the system to measure the polarization orientation. A simple telescope is used to form an image of the lamp which is captured by a 16-bit CCD camera with resolution 512 $\times$ 512 pixels.

Figure 8 shows a sequence of images obtained for half-wave-plate orientations between $-20^\circ$ and $20^\circ$ in $5^\circ$ steps. The effective polarization axis of the multiplet changes by twice the wave-plate rotation angle. The $k_x$ fringe pattern is due to the fixed optical delay providing spectral discrimination between the split $\pi$ and $\sigma$ components (see equation (19)). It has been confirmed that these fringes disappear when the field is zero or the primary delay plate is removed. The checkerboard fringe pattern displayed by the central image (corresponding to $0^\circ$ wave-plate rotation) gives $4\theta = 180^\circ$ corresponding to alignment of the multiplet parallel to the image $x$–$y$ axes ($\beta = 0$). The observed fringe contrast is around 25% and is set by both
Figure 9. Line profiles extracted from the image sequence shown in figure 8. Top: row 117 (amplitude encoding) and Bottom: row 108 (phase encoding). See text for discussion.

the achievable magnetic field strength and a somewhat poor instrument contrast because of the required large thickness of the primary wave plates.

The distortion of the fringes with effective rotation of the Zeeman multiplet agrees with theoretical expectations. To more clearly illustrate this point, figure 9 shows image line profiles obtained at rows 117 and 108 for which $\sin \phi_1 \approx 0$ and $\sin \phi_1 \approx \pi/2$, respectively, for wave-plate angles in the range $-30^\circ$ to $30^\circ$ in $5^\circ$ steps. The observed respective amplitude and phase modulation encoding of the tilt angle are in accord with the theoretical expectations of equation (17).

A simple algorithm has been developed to recover the polarimetric angle $\theta$ from the spatially modulated images. First note that the trigonometric term in equation (17) can be written as $A(y, \theta) \cos[\phi_1 + \psi(y, \theta)]$ with amplitude $A(y, \theta) = \sqrt{\cos^2 2\theta + \sin^2 2\theta \sin^2 \phi_1(y)}$ and y-varying phase given by $\tan \psi(y, \theta) = \tan 2\theta \sin \phi_1(y)$. It is assumed that the polarimetric angle $\theta(x, y)$ varies slowly compared with the carrier waves. Information about the angle $\theta$ is shared between the amplitude and phase terms, both of which can be recovered by demodulating the $k_x$ carrier. We have opted to extract the phase $\psi$ using a Morlet wavelet demodulation procedure [14]. Upon taking the tangent of $\psi$, it is straightforward then to demodulate the $k_y$ carrier for the amplitude $\tan 2\theta$. The full $\theta = 25^\circ$ image in figure 8 and its demodulated polarimetric angle image are shown in figure 10. The spatial variation of the polarization tilt angle across the image is likely due to refraction and back-reflection of light from the tube glass envelope.

6. Practical considerations for an imaging MSE spectro-polarimeter

As for other MSE polarimetric systems, it is convenient to physically separate the polarimetric and color selecting functions. For an imaging MSE system, the spectro-polarimeter and lenses reside close to the plasma and form an image of the interferogram onto an imaging fiber bundle for transport to the filter and camera (see figure 11). However, there are a number of special considerations for wide-FOV MSE imaging.
Figure 10. Left: Spectro-polarimetric image of the Zeeman lamp and right: the demodulated polarization angle $\theta$ (in degrees). The lamp image corresponds to the image labeled ‘25\(^\circ\)’ in figure 8. See text for discussion.

Figure 11. Diagram showing conceptual layout for imaging MSE system.

Firstly, the choice of fringe spatial frequency (Savart plate thickness) is a compromise between spatial resolution and loss of fringe contrast due to averaging over the pixel size, and this determines the number of fringes across the detector, typically in the range 50–100. From figure 7(b) it can be seen that due to the changing Stark separation $\Delta \lambda_S$, the optimum delay for maximum nett contrast varies approximately linearly across the FOV, spanning a range of approximately 100 waves. To track this variation, it is therefore advantageous to ensure that the direction of increasing fringe phase is in the same sense as the expected shear in optimum optical delay. To minimize distortion in wide-FOV applications, it may also prove valuable to employ compensated Savart plates as mentioned in appendix A.

Because of the change in Doppler shift $\Delta \lambda$ across the FOV, there will be an associated change in interferometric fringe phase of order $N \Delta \lambda / \lambda_0$ waves. It therefore makes sense to orient the optical system $x$-axis parallel to the projection of the beam direction so that the carrier waves are shifted in the $x$-direction with minimal $y$-distortion. In this case, the polarimeter fringes are aligned in the $y$-direction, parallel to $v \times B_T$, where $B_T$ is the toroidal magnetic field, and the beam and optical system $z$-axis are assumed to lie in the horizontal midplane. In
this case, injection into gas would deliver an interference pattern similar to that for the central image in figure 8 (\(\beta = \pi/2, \theta = 3\pi/4\)).

It is well known that the center wavelength of a bandpass interference filter shifts to the blue with increasing angle of incidence. This is a very important consideration for wide-angle imaging systems. After the imaging fiber array, one can use beam expansion optics to reduce the range of ray angles and tune a large aperture interference filter in such a way that the passband tracks the approximately 1 nm Doppler shift of the MSE cluster across the FOV. We emphasize, however, that this measure is to optimize the instrument performance and not essential for its function.

A future study will explore the expected angle signal-to-noise ratio as a function of nett contrast, fringe configuration, camera performance and other factors.

6.1. Self-calibration: recovering the full polarization state

It has been noted elsewhere [15] that it is possible to self-calibrate the polarization response of an MSE light collection system provided that one can measure the full Stokes vector of unpolarized emission. Because the MSE multiplet is, in principle, globally unpolarized, the provision of an unpolarized source can be ensured by removing the spectrally discriminating primary wave plate. Alternatively, one can retain the wave-plate and simply observe the plasma emission without beam.

It has been shown recently that a dual Savart plate imaging arrangement is sufficient to characterize the full Stokes vector description of a partially or fully polarized spectral scene [16]. The approach uses crossed Savart plates with intervening half-wave plate and a final analyzer to produce a two-dimensional fringe pattern that can be decoded for the four Stokes parameters. For a spectro-polarimetric imaging applications, one could arrange for the spatial carriers to have wave-vectors all with the same orientation (in this case, in the \(k_y\)-direction) but generating distinct spatial frequencies. For the purposes of MSE calibration, where temporal resolution is not an issue, one might alternatively choose to combine the spatial heterodyne system with a suitable combination of switchable or variable liquid crystal wave plates in order to capture the full Stokes vector [17].

In related spectro-polarimetric applications, such as Zeeman effect spectroscopy, it is also certainly desirable to capture the full polarization state. The utility of spatial heterodyne techniques for imaging of closely spaced complex Zeeman multiplet structures will be explored in a later paper.

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Appendix A

Here we review the construction and optical properties of various Savart prisms. A single birefringent plate (refractive indices \(n_e\) and \(n_o\)) cut with its fast axis oriented at 45° to the optical system \(z\)-axis in the \(x-z\) principal plane gives rise to an \(x\) displacement between the extraordinary and ordinary rays that exit the crystal. The optical path difference (OPD) between
the separated, orthogonally polarized waves is given by [18]

\[ \Delta = t \left( A + \frac{a^2 - b^2}{a^2 + b^2} \cos \omega \sin \theta_i + \cdots \right), \]

where \( a = 1/n_e, \ b = 1/n_o, \ t \) is the plate thickness,

\[ A = \left( \frac{2}{a^2 + b^2} \right)^{1/2} - \frac{1}{b}, \]

\( \theta_i \) is the angle of incidence and \( \omega \) is the angle between the plane of incidence and the principal section of the crystal plate. We see that the plate introduces a fixed phase delay and a phase shear proportional to the angle of incidence (for small angles). The lateral separation of the rays and its associated angle-dependent OPD gives rise to a pattern of interference fringes in the \( y \)-direction when imaged by a lens through an appropriately oriented analyzer.

A zero-nett-delay Savart shearing prism is formed by combining two such shear plates with their principal planes oriented orthogonally. The result of successive orthogonal displacements in the \( x \)- and \( y \)-directions produced by the double plate is a diagonal displacement as shown in figure 12. For a plate for total thickness \( 2t \), the OPD in this case is [18]

\[ \Delta = t \left( \frac{a^2 - b^2}{a^2 + b^2} (\cos \omega + \sin \omega) \sin \theta_i + \cdots \right), \]

\[ \approx d \theta_i \]  

where, in the approximate form, \( d \) is the nett spatial separation of the rays exiting the prism. For wide field of view, the higher order angle-dependent terms indicated in equation (30) can result in interference fringes deviating from straight and parallel. This can be overcome using field-widening compensation techniques described elsewhere [18]. With the group delay for the Savart plate given by equation (15), it is straightforward to show that the logarithmic derivative of the optical delay with respect to optical frequency is given by

\[ \frac{v_0}{\tau_0} \frac{\partial \tau}{\partial \nu} \bigg|_{v_0} = \frac{4a^2b^2(\beta - \alpha)}{a^4 - b^4}, \]

where

\[ \alpha = \frac{v_0}{n_o} \frac{\partial n_o}{\partial \nu} \bigg|_{v_0}, \]

\[ \beta = \frac{v_0}{n_e} \frac{\partial n_e}{\partial \nu} \bigg|_{v_0}. \]
References