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LETTERS

BALLOONING OPTIMIZED PRESSURE PROFILES IN TOROIDAL HELIACS

W.A. COOPER (Centre de Recherches en Physique des Plasmas, Association Euratom–Confédération Suisse, Ecole Polytechnique Fédérale de Lausanne, Lausanne, Switzerland)

H.J. GARDNER (Plasma Research Laboratory and Department of Theoretical Physics, Research School of Physical Sciences and Engineering, The Australian National University, Canberra, Australia)

ABSTRACT. The local 3-D ideal MHD stability properties for two different H-1 Heliac configurations with values of magnetic well of 1.3 and 3.2%, respectively, have been investigated. The pressure profile has been determined that approaches marginal stability to 3-D ballooning modes almost uniformly across the plasma radius for each case. In spite of the difference in well depths, the \( \beta \) limits for both configurations are almost the same (at \( \beta = 0.8\% \)). The optimal pressure profile for the deep well configuration is slightly more peaked than that for the standard configuration. The ballooning imposed \( \beta \) limits are more stringent than those of the Mercier criterion (\( \beta = 1\% \)) and the ballooning optimized pressure profiles are more peaked than the corresponding profiles which are marginal according to the Mercier criterion.

1. INTRODUCTION

Ideal magnetohydrodynamic (MHD) stability studies of Heliac devices have shown that the volume averaged \( \beta \) limits in the straight, helically symmetric, limit could reach values of 30% [1–3]. Investigations of local MHD stability in fully three dimensional (3-D) configurations, such as the evaluation of the 3-D Mercier criterion for the 3 period H-1 Heliac in operation in Australia [4, 5] and the 4 period TJ-II Heliac under construction in Spain [6–8], and the application of a 3-D ballooning mode equation solver to TJ-II [9] have demonstrated that the critical \( \beta \) limits are much lower in toroidal devices with a smaller number of field periods.

We have performed an analysis of 3-D ballooning [10, 11] and ideal Mercier modes for two different configurations that are attainable with the coil system of the H-1 Heliac [4]. Our ‘standard configuration’ has no current in the helical trim-coil that is wound around the central conductor and has a 1.3% magnetic well. The ‘deep well’ case, with a 3.2% magnetic well, has a helical coil current that is 8% of that in the central conductor [5]. The Mercier stability properties of these two configurations have been analysed in detail by Gardner and Blackwell [5], who investigated the parallel-current-density drive mechanism for instability and demonstrated that in both cases a plasma mass profile that is almost linear in the radial flux-variable, \( s \), approached marginal stability across the plasma radius except in the vicinity of the \( \tau_p = 3/8 \) resonance in the standard configuration and the \( \tau_p = 4/9 \) resonance in the deep well configuration. Here, \( \tau_p \) corresponds to the rotational transform per field period. The pressure profiles have to be flattened around the resonant surfaces in question in order to guarantee stability.

We have extended the investigation of Gardner and Blackwell to include ballooning modes for each case. For each configuration we adjusted the pressure profiles until the approach to marginal stability was almost uniform across the plasma cross-section. In the evaluation of the local MHD stability of H-1, we employed the magnetic (indirect) method to determine the parallel current density. This method yields more reliable estimates of the Mercier stability [5] than the geometric (direct) method, which tends to smooth out the effects of resonant surfaces. As found in a recent study for the W7-X stellarator [12], however, the ballooning eigenmodes for H-1 (being more localized along a magnetic field line than Mercier modes) do not appear to depend sensitively on the method used.

2. 3-D EQUILIBRIUM AND LOCAL STABILITY

We have computed the 3-D MHD equilibrium state using the VMEC code [13], which minimizes the energy functional

\[
W = \int \int d^3x \left( \frac{B^2}{2} + \frac{p(s)}{\Gamma - 1} \right)
\]

where \( B \) is the magnetic field that is assumed to consist of the nested magnetic flux surfaces which are labelled by the normalized, radial flux co-ordinate, \( s \) (ranging from \( s = 0 \) at the magnetic axis to \( s = 1 \) at the boundary). The plasma pressure is given by

\[
p(s) = \frac{M(s)}{[V'(s)]^\Gamma}
\]

where \( M(s) \) is the invariant plasma mass function, \( V'(s) \) is the differential plasma volume and \( \Gamma \) is the adiabatic index. The prime denotes the derivative of a flux surface quantity with respect to \( s \). In this Letter, we have chosen to specify \( \Gamma = 2 \). In VMEC, two surface functions are prescribed. One is the net toroidal
plasma current enclosed within each flux tube, which we chose to be zero, and the other is given by
\[
m(s) = \frac{M(s)}{[\Phi^*(s)R_b]^2}
\]
where \(2n^+(s)\) is the toroidal magnetic flux enclosed within \(s\) and \(R_b\) is the average of \(R\) (the distance from the major axis) around the plasma-vacuum interface boundary.

The 3-D ballooning mode equation [10, 11] can be expressed in the form
\[
\frac{\partial}{\partial \theta} \left( [C_p + C_3(\theta - \theta_b)] \frac{\partial X}{\partial \theta} \right) + (1 - \lambda)(d_p + d_3(\theta - \theta_b))X = 0
\]
(4)
in Boozer magnetic co-ordinates [14]. Negative values of the eigenvalue, \(\lambda\), correspond to stability. Detailed forms for the coefficients can be found in Ref. [15]. Retaining the ballooning eigenvalue in an asymptotic analysis of Eq. (4), we obtain a form of the Mercier criterion where the value of \(1 - \lambda\) constitutes a measure by which the pressure gradient deviates from the critical pressure gradient for marginal stability.

3. STABILITY RESULTS

The H-1 Heliac equilibrium calculations were carried out in the fixed boundary mode of VMEC using a total of 176 Fourier amplitudes in \(R\) and \(Z\) to describe the plasma boundary. In order to obtain sufficiently high quality equilibria to analyse for local ideal MHD stability, we found it necessary to employ a mode selection pattern for the equilibrium runs that encompassed \(0 \leq m \leq 9\) and \(-16 \leq n/L \leq 16\), where \(m\) is the poloidal mode number, \(n\) is the toroidal mode number and \(L\) is the number of field periods (3 for H-1). The number of poloidal mesh points was \(N_p = 32\) and the number of toroidal mesh points was \(N_t = 48\). The force-tolerance convergence parameter in VMEC was held fixed at \(f_{tol} = 5 \times 10^{-11}\). Before undertaking the stability analysis, we reconstructed, in VMEC co-ordinates, the periodic poloidal angle renormalization function [16] by solving the condition \(j \cdot \nabla s = 0\) on each flux surface, where \(j\) is the current density. In order to do this, two sidebands, both poloidal and toroidal, were added to the VMEC mode selection pattern. For the equilibrium reconstruction in Boozer co-ordinates [14], we added the modes \(12 \leq m \leq 30\) and \(-4 \leq n - m_p \leq 4\) to the mode pattern previously selected. We increased the angular grid, correspondingly, to \(N_p = 124\) and \(N_t = 72\). This mode pattern was adequate for the accurate reconstruction of equilibria in the Boozer co-ordinate system up to \(\beta = 1.5\%\). Mode selection patterns that are comparably broad have been required to resolve ballooning mode structures in the TJ-II Heliac [9]. To determine the optimal ballooning mode profiles, we initially calculated equilibria on a 24 interval radial mesh and iterated the shape of the input mass profile until near marginal stability to ballooning modes was achieved throughout most of the plasma. We then fine tuned the calculations on a 96 interval radial mesh. It seems reasonable to assume that computing structures with a

![Fig. 1](image1.png)

**FIG. 1.** Ballooning mode eigenvalues as functions of \(s\) at \(\beta\) values of 0.72, 0.782, 0.85 and 0.907% from bottom to top, respectively, for the standard H-1 configuration with an optimized pressure profile.

![Fig. 2](image2.png)

**FIG. 2.** Ballooning mode eigenvalues as functions of \(s\) at \(\beta\) values of 0.717, 0.785, 0.853 and 0.920% from bottom to top, respectively, for the deep well H-1 configuration with an optimized pressure profile.
much finer radial resolution than this would test the validity of the ideal MHD model. Vanishing boundary conditions were imposed on the ballooning eigenfunction at the end points of 15 poloidal transits of each magnetic field line. The midpoint corresponded to the location where the curvature was most destabilizing.

The radial profiles of the ballooning eigenvalue, \( \lambda \), at four different values of \( \beta \) for the standard configuration is shown in Fig. 1. The prescribed plasma mass profile is

\[
m(s) = m_0 \left( 1 - s + \frac{1}{15} (1 - s)^2 + \frac{1}{3} (1 - s)^3 \right)
\]

(5)

The values of \( m_0 \) are 0.736, 0.8, 0.864 and 0.928, respectively, from bottom to top (most stable to most unstable). These results show that the ballooning imposed stability limit is around \( \beta = 0.8\% \). The radial profiles of \( \lambda \) at four different values of \( \beta \) for the deep well configuration are shown in Fig. 2. The plasma mass profile is given by

\[
m(s) = m_0 \left( 1 - s + \frac{13}{121} (1 - s)^2 + \frac{1}{3} (1 - s)^3 \right)
\]

(6)

The values of \( m_0 \) in this configuration are 0.63, 0.69, 0.75 and 0.81, respectively, from the most stable to the most unstable case. We also find a \( \beta = 0.8\% \) limit in the deep well configuration. The shape of the optimal pressure profiles imposed by 3-D ballooning modes for the two H-1 configurations under consideration are shown in Fig. 3, where it can be seen that the profile in the deep well configuration is slightly more peaked than that of the standard configuration and both are more peaked than the linear profile.

In Fig. 4, we show the value of the Mercier criterion (using the form in which we have retained \( \lambda [12, 15] \)) for the marginally stable standard H-1 configuration at \( \beta = 0.782\% \). The Mercier criterion predicts stability everywhere except in a region that constitutes about 5% of the plasma volume near \( s = 0.4 \), where the resonant surface \( t_p = 3/8 \) is located. There is also a narrow unstable region near the plasma edge where the \( t_p = 5/13 \) resonant surface appears. We anticipate that some local flattening of the pressure profile in the vicinity of these critical surfaces would guarantee stability without significant changes in the \( \beta \) limit predicted by ballooning modes. The radial profile of the Mercier criterion for the marginally ballooning-stable deep well configuration (at \( \beta = 0.785\% \)) is shown in Fig. 5. Here the plasma is stable except at the edge where the \( t_p = 4/9 \) resonant surface is located. In this case, we would except an island structure about the 4/9 resonance, which might reduce the effective size of the plasma by about 5%. We note that there are two additional spikes in Fig. 5, which correspond to higher order rational surfaces. The spike at \( s = 0.6 \) corresponds to the \( t_p = 7/16 \) resonant surfaces and the spike at \( s = 0.75 \) corresponds to the \( t_p = 11/25 \) resonant surface. The structures around these surfaces (as well as those at the \( t_p = 10/23 \) surface which we did not detect) are, presumably, so localized radially that they constitute a fraction of a typical fusion-plasma thermal-ion Larmor radius so that the ideal MHD model would not apply. In other words, our interpretation of the resonances in the Mercier criterion is that pressure profile flattening will occur about low
order rational surfaces but that the consideration of high order resonances is beyond the bounds of our model.

4. DISCUSSION

The ballooning eigenvalue curves approach the marginal point quite uniformly, as a function of the radial variable, with the mass profiles we have identified for the standard and deep well configurations except for a region near the magnetic axis. The variation of the coefficients of the ballooning mode equation is very weak along field lines in the vicinity of the centre of the plasma, making it difficult to resolve correctly structures within a domain that spans merely 15 poloidal transits. Consequently, we do not consider a detailed optimization of the pressure profiles in the inner 10% of the plasma volume to be necessary. Furthermore, the magnetic axis constitutes a singular point in the flux co-ordinate system where it becomes difficult to compute equilibrium quantities accurately.

The localized ballooning structures are insensitive to the singularities in the parallel current density at rational surfaces, thus avoiding the problems of interpretation that have affected the application of the Mercier criterion. For example, the Mercier stability study of Ref. [5] found a critical $\beta$ of 0.95% for the standard configuration and 1.3% for the deep well configuration. These estimates used the magnetic method for calculating the parallel current density. However, these critical $\beta$s were increased to 1.3 and 1.9%, respectively. On the other hand, despite the large difference in the values of the magnetic well of the two configurations, the difference in the $\beta$ limits for ballooning stability is very small. Furthermore, the ballooning stability estimates do not need to be buttressed by caveats about the manner in which parallel currents have been treated. Our ballooning stability estimates support the view that local variations of the shear and of the magnetic curvatures dominate over flux surface averaged quantities, such as the global shear and the magnetic well for 3-D ballooning modes in low shear stellarators with relatively small aspect ratios and a small number of field periods. (A similar result has recently been found for drift waves [17].) An important component of the experimental program of H-1 will be to determine whether increasing the magnetic well leads to any increase in MHD stability. On this subject, we note that the great flexibility of the TJ-II Heliac, which allows it to realize configurations with nearly fixed rotational transform but extremely variable magnetic wells, may hold the promise of being able to elucidate the relevant physical mechanisms responsible for instabilities of this class.

As a postscript on the subject of numerical accuracy, we note that the VMEC code uses a multi-grid approach to determine the minimum energy state [13] which reduces computation time but may lead to the appearance of fictitious resonances if a coarse precursor grid is used. For the fine grid computations method for calculating the parallel current density increased these critical $\beta$s to 1.3 and 1.9%, respectively. On the other hand, despite the large difference in the values of the magnetic well of the two configurations, the difference in the $\beta$ limits for ballooning stability is very small. Furthermore, the ballooning stability estimates do not need to be buttressed by caveats about the manner in which parallel currents have been treated. Our ballooning stability estimates support the view that local variations of the shear and of the magnetic curvatures dominate over flux surface averaged quantities, such as the global shear and the magnetic well for 3-D ballooning modes in low shear stellarators with relatively small aspect ratios and a small number of field periods. (A similar result has recently been found for drift waves [17].) An important component of the experimental program of H-1 will be to determine whether increasing the magnetic well leads to any increase in MHD stability. On this subject, we note that the great flexibility of the TJ-II Heliac, which allows it to realize configurations with nearly fixed rotational transform but extremely variable magnetic wells, may hold the promise of being able to elucidate the relevant physical mechanisms responsible for instabilities of this class.

As a postscript on the subject of numerical accuracy, we note that the VMEC code uses a multi-grid approach to determine the minimum energy state [13] which reduces computation time but may lead to the appearance of fictitious resonances if a coarse precursor grid is used. For the fine grid computations
we have presented here, a precursor run with 38 radial grid points was employed. The effect of varying this can be seen in Fig. 6, where we show the Mercier criterion for the deep well configurations at $\beta = 0.72\%$ and $\beta = 0.92\%$ (where the different $\beta$ values are plotted for the sake of clarity as well as to form a set with Fig. 5). Whereas both computations were carried out on a radial mesh of 96 surfaces, the smoother, dashed, curve was generated without a precursor run and the solid curve corresponds to a precursor run with 12 grid points. The bumpiness of the solid curve is related to the positions of the grid points in the precursor run and indicates the presence of residual errors in the equilibrium solution. The precursor runs with 38 grid points (such as in Fig. 5) reduced the magnitude of these wiggles to manageable levels, while speeding up the computation of the equilibrium state. (Note that the mode selection pattern chosen for this comparison was slightly smaller than for the main study and excluded the $iP = 11/25$ resonance and, also, that the change in rotational transform with resonance in from the plasma edge.)

5. CONCLUSIONS

We have investigated the local 3-D ideal MHD stability of the standard configuration and a deep magnetic well configuration of the H-1 Heliac. We varied the mass profile in each case until we determined the conditions where the plasma became unstable to ballooning modes nearly uniformly across the plasma radius. The ballooning optimized pressure profile in the deep well configuration is slightly more peaked than that in the standard configuration and both are more peaked than the corresponding Mercier optimized profiles. The variation of the magnetic well from 1.3 to 3.2\% did not appear to improve the critical $\beta$ at all. The ballooning $\beta$ limit at $\beta = 0.8\%$ in both configurations is more stringent than that predicted by the Mercier criterion by about 20\%.

We anticipate that our predictions for the $\beta$ limits and the shapes of the pressure profiles (marginal to Mercier or to ballooning modes) can be tested in the H-1 experiment. Such comparisons will be important for evaluating the applicability, reliability and relevance of local ideal MHD stability theory and computation to the design of future 3-D stellarator devices.

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